DO NOW

Integrate: $\int \frac{x^2+x+1}{x^2+1}\,dx$

$$\int 1\,dx + \int \frac{x}{x^2+1}\,dx$$

$$\int 1\,dx + \frac{1}{2}\int \frac{2x}{x^2+1}\,dx \qquad u = x^2+1,\ du = 2x\,dx$$

$$\boxed{x + \tfrac{1}{2}\ln|x^2+1| + C}$$

$$x^2+1 \overline{)\,x^2+x+1}\ \Rightarrow\ 1 + \frac{x}{x^2+1},\quad \text{remainder } x$$

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5.7 The Natural Logarithmic Function: Integration - Day 2

Guidelines for Integration:

1. Learn the basic list of integration formulas.
2. Use *u*-substitution to rewrite the integrand so it conforms to one of the formulas.
3. If *u*-substitution does not work, try altering the integrand. Try trigonometric identities, multiplication/division/addition/subtraction of the same quantity, long division....Be creative.
4. Try using computer software (calculators) to find antiderivatives.

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Examples:

1. $\int \frac{2x}{(x+1)^2}\,dx$

$u = x+1 \Rightarrow x = u-1$, $du = dx$

$$\int \frac{2(u-1)}{u^2}\,du$$

$$\int \frac{2u-2}{u^2}\,du$$

$$\int \left(\frac{2u}{u^2} - \frac{2}{u^2}\right)du$$

$$2\int \frac{1}{u}\,du - 2\int u^{-2}\,du$$

$$2\ln|u| - 2(-u^{-1}) + C$$

$$\boxed{2\ln|x+1| + \frac{2}{x+1} + C}$$

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In the basic derivatives, there is nothing where the derivative is tan *x*....

2. $\int \tan x\,dx$

$$\int \frac{\sin x}{\cos x}\,dx \qquad u = \cos x,\ du = -\sin x\,dx$$

$$-\int \frac{-\sin x}{\cos x}\,dx$$

$$\boxed{-\ln|\cos x| + C}$$

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3. $\int \sec x\,dx$

$$\int \sec x\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)dx$$

$$\int \frac{\sec^2 x + \sec x\tan x}{\sec x + \tan x}\,dx \qquad u = \sec x + \tan x,\ du = (\sec x\tan x + \sec^2 x)\,dx$$

$$\boxed{\ln|\sec x + \tan x| + C}$$

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Integrals of the Six Basic Trigonometric Functions

$$\int \sin u\,du = -\cos u + C$$

$$\int \cos u\,du = \sin u + C$$

$$\int \tan u\,du = -\ln|\cos u| + C$$

$$\int \cot u\,du = \ln|\sin u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc u\,du = -\ln|\csc u + \cot u| + C$$

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4. $\int \cot 3x\, dx$

$u = 3x$
$du = 3dx$

$\frac{1}{3}\int \cot u\, du$

$\frac{1}{3}\ln|\sin u| + C$

$\frac{1}{3}\ln|\sin 3x| + C$

5. $\int_0^{\frac{\pi}{4}} \sqrt{1+\tan^2 x}\, dx$

$\int_0^{\pi/4} \sqrt{\sec^2 x}\, dx$

$\int_0^{\pi/4} \sec x\, dx$

$\left[\ln|\sec x + \tan x|\right]_0^{\pi/4}$

$\ln|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}| - \ln|\sec 0 + \tan 0|$

$\ln|\sqrt{2} + 1| - \ln|1 + 0|$

$\ln|\sqrt{2}+1|$

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HOMEWORK

pg 358 - 359; 19, 21, 23, 29 - 38,
49 - 55 odd, 69, 73